

WHY THE REAL PART OF THE PROTON-PROTON FORWARD SCATTERING AMPLITUDE SHOULD BE MEASURED AT THE LHC ^a

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Abstract

For the energy of 14 TeV, to be reached at the Large Hadron Collider (LHC), we have had for some time accurate predictions for both the real and imaginary parts of the forward proton-proton elastic scattering amplitude. LHC is now scheduled to start operating in two years, and it is timely to discuss some of the important consequences of the measurements of both the total cross-section and the ratio of the real to the imaginary part. We stress the importance of measuring the real part of the proton-proton forward scattering amplitude at LHC, because a deviation from existing theoretical predictions could be a strong sign for new physics.

We all know that, up to now, scattering amplitudes of PHYSICAL, strongly interacting particles (*i.e.* baryons and mesons) appear to satisfy dispersion relations, while in the particular case of quarks, since there are no asymptotic states, this statement would be meaningless. The most general versions of local quantum field theory lead to proving dispersion relations¹ and, more generally, analyticity properties in two variables in a rather large domain, if one makes use of the positivity properties of the absorptive part of the scattering amplitude². Furthermore one can prove the bound for the total cross section

$$\sigma_{total} < \text{Const}(\log s)^2, \quad (1)$$

where s is the square of the center of mass (c.m.) energy.

The first question one faces regarding the above results is the composite nature of protons and mesons, *i.e.* their quark-gluon structure. Some physicists doubted that composite particles could be described by local fields. However, Zimmermann has proved long ago that a local field operator could be used as an interpolating field to represent a composite particle³. The asymptotic free *in* and *out* limits of this field, could then be used to obtain S-matrix elements involving composite objects like a proton. In the sixties, it was realized that asymptotic theory, reduction formulae and standard analyticity properties hold also when particles, in particular composite particles, are created by polynomials in regularized local fields or local observables, acting on the vacuum. It was also shown in Ref. 1, that even in these cases, the scattering amplitudes are polynomially bounded, so that dispersion relations hold in the same way as for strictly local fields. We shall take this picture as our starting point.

Empirically, over many years, dispersion relations have always been consistent with the measured data for energies reached by fixed target machines (*e.g.* pion nucleon scattering) or colliders (ISR and SP $\bar{\text{P}}$ S colliders). Unfortunately the measurement of ρ (see Eq.(2)) at

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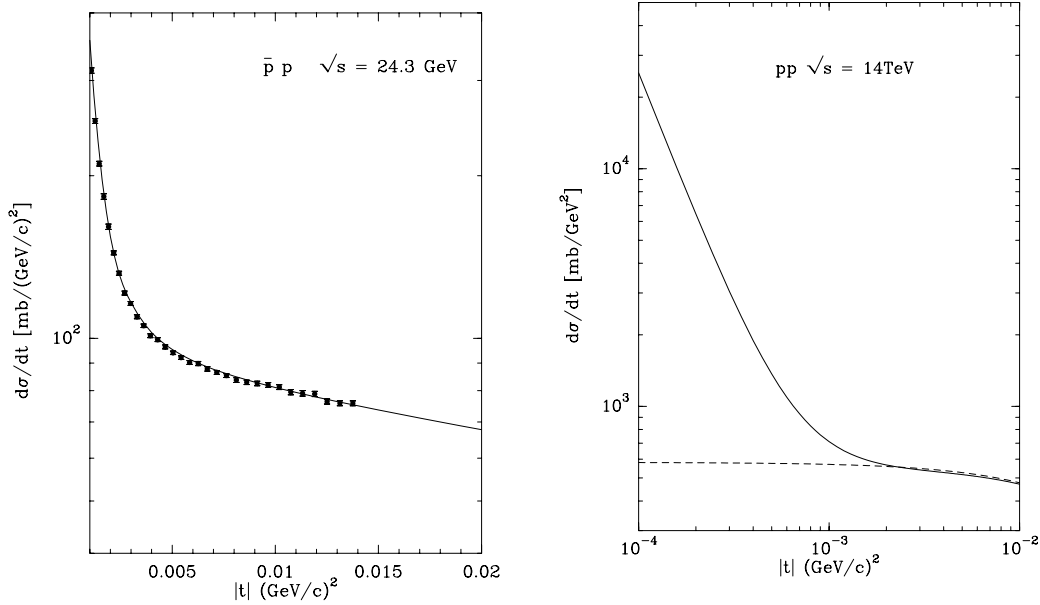


Figure 1: Predictions from the BSW model. On the left, $d\sigma/dt$ for $\bar{p}p$ as a function of $|t|$ in the small t region for $\sqrt{s} = 24.3\text{GeV}$ compared to the UA6 data⁷ (Taken from Ref. [5]). On the right, $d\sigma/dt$ for pp elastic scattering at the LHC energy, as a function of $|t|$ in the small t region. The dashed curve is the pure hadronic contribution, while the solid curve includes both the hadronic and the Coulomb amplitudes.

the Tevatron has such large errors that, no useful information can be extracted from the data.

The question of what will happen at LHC energies is completely open. Here \sqrt{s} will be 30 times higher than the highest energy for which ρ has been previously accurately measured. From the point of view of some string theorists extra dimensions that are needed for string theory, could be larger than the compact ones proposed in the early string days. Indeed, in some recent models⁴, these could be of a scale not far above that of LHC and could introduce observable non-local effects. However, one should note that at present, none of these string type theories have a clear definition of a scattering amplitude.

If $a(s, t)$ denotes the spin-independent amplitude for pp (and $\bar{p}\bar{p}$) elastic scattering, where t is the momentum transfer, we define the ratio of real to imaginary parts of the forward amplitude

$$\rho(s) = \frac{\text{Re } a(s, t=0)}{\text{Im } a(s, t=0)} , \quad (2)$$

the total cross section

$$\sigma_{total}(s) = \frac{4\pi}{s} \text{Im } a(s, t=0) \quad (3)$$

and the differential cross section

$$\frac{d\sigma(s, t)}{dt} = \frac{\pi}{s^2} |a(s, t)|^2 . \quad (4)$$

We recall that three of us, Bourrely, Soffer and Wu (BSW), have proposed more than twenty years ago an impact picture approach⁵, based on the work of Cheng and Wu⁶, which describes accurately all available pp and $\bar{p}\bar{p}$ elastic data. Several predictions have been made and, as an illustration, we show in Fig. 1 (left), the predicted cross section for $\bar{p}p$, in the Coulomb Nuclear Interference (CNI) region, compared to the UA6 data⁷.

Let us now come back to the important question of testing dispersion relations which can be done in two ways :

- use an explicit model which reproduces very well all existing data, and satisfies, by construction, dispersion relations, such as the BSW model.

- use fits of existing data, *e.g.* the one performed by the UA4/2 Collaboration⁸.

The superiority of the first approach is that there is no flexibility in the predictions, while in the second it is essential to take a smooth fit, depending on a few parameters, because otherwise, the predictive power is lost. At the LHC energy $\sqrt{s} = 14\text{TeV}$, the BSW model predicts

$$\rho = 0.122 \quad \text{with} \quad \sigma_{total} = 103.6\text{mb} \quad (5)$$

and for completeness we display in Fig. 1 (right), the predicted cross section in the very small t region. The UA4/2 fit predicts

$$\rho = 0.13 \pm 0.018 \quad \text{with} \quad \sigma_{total} = 109 \pm 8\text{mb}. \quad (6)$$

We see that these numerical predictions are compatible. If the experiment gives numbers compatible with those above, it will mean that the scale of violation is very much above the LHC energy or, that the corresponding minimal size is much smaller. It will also mean that the predicted cross sections of the model and of the fit are valid at much higher energies. This will allow us to have a better idea about the magnitude of the cross sections at energies which might never be accessible, except by cosmic ray experiments.

The next question is: what can one conclude if the real part of the amplitude obtained from the dispersion integral over σ_{total} turns out to be significantly different from the measured one? There are three possible conclusions, all indicating new physics, that would result from such a disagreement. First, it could be that the total cross section beyond the LHC energy region is radically different from what we now believe, based on the indications coming from cosmic ray data or our expectation of a smooth slow logarithmic growth for σ_{total} . This would be a very important signal for new physics. Second, it is quite possible, though less-likely, that in the gap we are faced with, between 0.5 TeV and 14 TeV, something unexpected is happening, *e.g.* one or more resonances or significant changes in σ_{total} , which is again new physics. This "gap" was imposed on physicists by the fear that LHC would be the last machine, and one had to go from $\sqrt{s} = 2$ TeV at the Tevatron, to $\sqrt{s} = 14$ TeV at LHC. This fact makes the failure of the Tevatron ρ -measurement more significant.

Thirdly, there is the possibility that dispersion relations themselves do not hold. This would be a very significant result.

Due to the fact that no violation is seen at lower energies, the violation must be progressive and it turns out, in our proposal as we will see, that the violation is controlled by a single parameter. We assume that the initial analyticity domain obtained by local field theory (without the extension due to positivity, which needs polynomial boundedness) is still valid, but that polynomial boundedness is violated in unphysical, in particular complex, regions of the analyticity domain⁹. It is not so easy to implement this violation, and for instance, if one assumes a growth like $\exp(\sqrt{s})$ in complex directions, one falls back, in the end on a polynomial bound, back on ordinary dispersion relations and back on the standard bound on the total cross section. The first case of non trivial violation of dispersion relations is when the scattering amplitude is allowed to behave like $\exp(s/s_0)$ in unphysical and/or complex directions. If we assume that this bound also holds inside the "Lehmann ellipse" at fixed energy, we can prove, using unitarity, that the physical amplitudes, for $t \leq 0$, are bounded by s^4 . This means that we can write a dispersion integral with four subtractions. However this is not the scattering amplitude which differs from the dispersion integral by an entire function of order one. Another way is to multiply the scattering amplitude by a convergence factor, which guarantees that the modified amplitude has no exponential growth in complex directions. Such a factor is

$$\exp(-\sqrt{(4m^2 - s)}\sqrt{(4m^2 - u)}/s_0), \quad (7)$$

a crossing symmetric term, where u is the third Mandelstam variable and m is the proton mass. Because of the fact that the real part of the scattering amplitude is indeed small in existing data, as well as in models and fits, the effects of this exponential growth are very

visible even if the scale of the exponential, s_0 , is much higher than the square of the LHC energy. For instance at LHC ($\sqrt{s}=14$ TeV), as stated above, we expect naively :

$$\rho = 0.12 \text{ to } 0.13. \quad (8)$$

With a scale $\sqrt{s_0} = 50\text{TeV}$, the modified amplitude would lead to

$$\rho = 0.21. \quad (9)$$

This means that we do not even need a very accurate measurement of ρ to see an effect. A measurement with 30% accuracy could be enough. This is why we are delighted that ρ will be measured by the ATLAS detector at CERN¹⁰, as a by-product of a luminosity measurement using the Coulomb interference region.

In closing we should stress the following: an experimental measurement of ρ giving a result consistent with Eqs. (5) and (6) will also be an important result and there will be no indication for new physics. However, we would then have an empirical test of local field theory at length scales 30 times smaller than what is presently known.

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References

1. H. Epstein, V. Glaser and A. Martin, *Comm. Math. Phys.* **13**, 257 (1969).
2. A. Martin, *Nuovo Cimento* **42**, 930 (1966) and **44**, 1219 (1966).
3. W. Zimmermann, *Nuovo Cimento* **10**, 597 (1958).
4. I. Antoniadis, *Eur. Phys. J. C* **33**, 914 (2004).
5. C. Bourrely, J. Soffer and T.T. Wu, *Phys. Rev.* **D19**, 3249 (1979), *Nucl. Phys. B* **247**, 15 (1984) and *Eur. Phys. J. C* **28**, 97 (2003).
6. H. Cheng and T.T. Wu, *Phys. Rev. Lett.* **24**, 1456 (1970).
7. UA6 Collaboration, R.E. Breedon *et al.*, *Phys. Lett. B* **216**, 459 (1989).
8. UA4/2 Collaboration, C. Augier *et al.*, *Phys. Lett. B* **316**, 448 (1993).
9. N. N. Khuri, Proceedings of Les Rencontres de Physique de la Vallée d'Aoste: Results and Perspectives in Particle Physics (M. Greco, ed.), pp 701-708, *Editions Frontières*, Gif-sur-Yvette, France, 1994.
10. M. Haguenaue, private communication.
See also, Atlas Collaboration, I. Efthymiopoulos, these proceedings.